STRIKING RESULTS WITH BOUNCING BALLS

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ABSTRACT
In a laboratory activity students study the behaviour of a bouncing ball. With the help of a high-speed camera they can study the motion in detail. Computer modelling enables them to relate the measurement results to the theory. They experience that reality (measurements) is not automatically in line with the predictions of the theory (the models), but often even strikingly apart. This stimulates a process of repeated cycles from measurement to interpretations (how to adapt the model?), and in this way it realizes a rich and complete laboratory activity. The activity is made possible by the integrated ICT tools for measurements on videos made by a high-speed camera (via automated point-tracking), and for modelling and simulations.

INTRODUCTION

Each introductory physics textbook, already at secondary school level, illustrates Newton's laws of motion and concepts of gravitational energy and kinetic energy with examples of objects dropped or thrown vertically and contains investigative activities about falling objects. The reasons are obvious:
- the physics and mathematics is still simple enough to be accessible to most of the students;
- an experiment with a falling object, in which data are collected with a stopwatch and meter stick, using sensors such as a microphone or a sonic ranger (MBL), or via webcams and video analysis tools (VBL), is easy to perform;
- it is a clear invitation to compare measurements with theoretical results.

Falling with air resistance is a natural extension of a free fall study. In this case, explorations are often directed towards two observable phenomena: (1) a falling object reaches a terminal velocity and (2) more massive objects fall faster than less massive objects. Experience from daily life already learns that air drag cannot be neglected in many practical cases, e.g., not for the motion of a table tennis ball. Important factors that have an effect upon the amount of air drag are the speed and cross-sectional area of the falling object. In popular experiments of dropping coffee filters (Derby *et al*, 1997), balls and party balloons (Gluck, 2003), or paper cones (Wooning *et al*, 2006), students investigate the movement of an object released at a certain height and they determine the influence of weight, size and shape of the falling object on its motion. Data collection is typically done by stopwatch and meter stick, it is MBL-based using a sonic ranger (Gluck, 2003; Wooning *et al*, 2006), or it consists of video recording with a digital camera or web cam, followed by measurement in a video analysis system (Pagonis *et al*, 1997; Mooldijk and Savelsbergh, 2000; Cross, 2007). Computer modelling allows the students to compare reality (measurements) with theory-based prediction (the models). However, a practical investigation of various models of air resistance is often omitted because it is in practice not always easy to find conclusive evidence of the value or nature of the resistive force: will the drag force acting on an object be proportional to the speed of the object, the square of the speed, or depend differently on the speed? The textbook or the teacher just gives the mathematical relationship for the drag force. However, with the advent of affordable high-speed technology, e.g., the release in 2009 of the Casio EXILIM FC-100

camera (price ≈ 300 euros), and by using the video tool in the Coach 6 computer learning environment (benefiting in particular from automated point-tracking and perspective correction facilities) this kind of practical investigations are brought within reach of high school students and students can explore themselves the mathematical relationship between drag force and speed (Heck & Uylings, 2010).

We report in this paper about a sequence of activities for pre-university students dealing with the topic of a bouncing ball. Focus is on mathematical modelling by students using principles of classical mechanics and on validation of the constructed models. We discuss not only an integrated application of mathematics and physics, but also the use of integrated ICT tools, which are part of the versatile learning and authoring environment called Coach (Heck *et al*, 2009) and which support construction and validation of models. The main aims of the developed activities are (cf. Hestenes, 1996):
1. By engaging secondary school students in modelling activities, give them a feel for what models and modelling mean in the practice of mathematicians and physicists;
2. By focusing on basic models, make the structure and coherence of scientific knowledge more evident to students.

The activities are supposed to increase the modelling competences of the students. Herein we take a holistic view on modelling processes in science and science education, and on modelling by students. We are of opinion that students learn best about modelling by actually doing modelling in authentic contexts, simple ones at first but complete modelling work nonetheless, becoming more sophisticated as confidence and experience increase. Parts of their work resemble more and more the processes of scientific practitioners in modelling. Because most scientists do not consider modelling as an alternative to empirical science and because they usually put great effort in validation of their models, we let students act in the same way and we always require a comparison of the results of a model with the empirical measurements. Thus, we pay in our activities a lot of attention to the validation of the (intermediate) models. Our goal is to promote a critical attitude of students by looking at various models of one and the same phenomenon. We want to go so far that the outcomes of model of a bouncing table tennis ball matches empirical measurements of the motion of the bouncing ball very well, i.e., we are not satisfied with just a fair prediction of a small part of the bouncing process or of the number of bounces, but we really request a very good match between the measured position-time graph and the graph produced by the computer model. As we will see this is not as easy as might be thought and a high-speed camera will be useful to study the motion of a bouncing table tennis ball in detail.

THE MODELLING CYCLE: MODEL OF MODELLING + GUIDELINE FOR INSTRUCTION

There are various views on scientific modelling and how students can develop modelling competences (see, for example, van de Berg *et al*, 2006). But, as Doerr and Pratt (2008) pointed out, the collective epistemological perspective is that "modelling is driven by the need to describe, explain, and/or predict some particular phenomena of interest to the modeller. Elements from the real world of the experienced phenomena are selected, organized, and structured in such a way that they can be mapped onto a model world. This model world necessarily simplifies and distorts some aspects of the real world while maintaining other features and allowing for manipulations of these features (or objects) in accordance with the rules of the model world." In other words, the model is separate from the world to be modelled. In this view, a model is a constructed system of objects, relationships, and rules whose behaviour resembles that of some other system in the real world of phenomena. The rules of the model world come in our type of modelling from mathematics and physics.

Another common epistemological underpinning to scientific modelling is that it is considered a cyclic or iterative process. Doerr and Pratt (2008) brought forward that "the source of this iteration comes from the attempts at validation in the real world of the outcomes of the manipulations of the objects in the model world. This validation can take several different forms. In some cases, the validity of outcomes is simply measured against the criteria of usefulness for some particular purpose. In other cases, validity is determined by comparison to other models or to other experienced phenomena or to predicted data. The outcomes of the validation process result in either a satisfactory model or generate

another cycle of modelling activity. The cyclic nature of this modelling paradigm is generally presented in variations of the simplified form suggested in Figure 1."
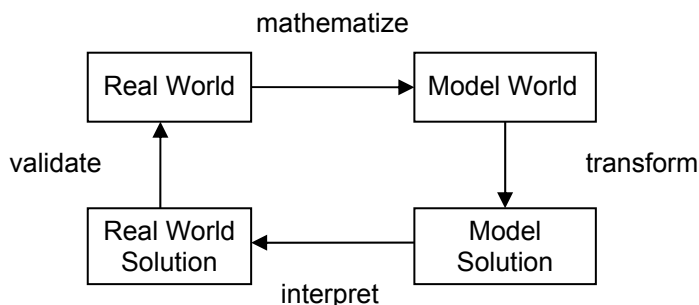

Figure 1. The cyclic nature of the modelling process in a nutshell.

One of the variations of the simplified modelling cycle, which we adopt in our work and which is quite popular in the research community on mathematics education, comes from Blum and Leiß (2005) and distinguishes seven sub-processes in the modelling process (Figure 2).
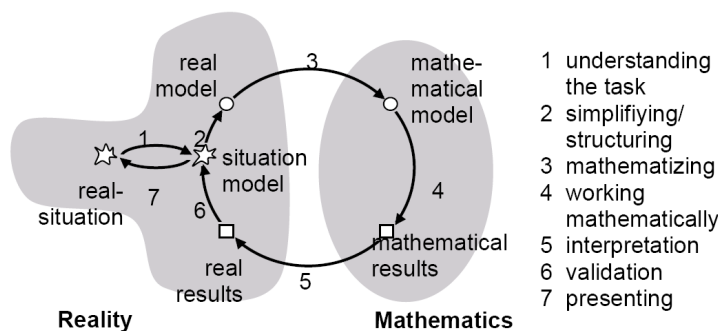

Figure 2. The modelling cycle of Blum and Leiß.

Such schemes are mostly used as theoretical framework for educational research to describe the 'ideal modelling process', to formulate modelling competencies, and to discuss obstacles of students during the distinct modelling phases and transitions (see, for example, Gallbraith and Stillman, 2006; Maaß, 2006). The schemes do not imply that one necessarily steps during modelling through each and every phase in the modelling cycle in the given ordering; modelling is in practice a non-linear process. For example, a simulation with a given computer model may form the starting point of an investigation to come to grips with the phenomenon to be modelled and may be used to brush up or extend the required mathematics and physics knowledge and skills. Blomhøj and Jensen (2003) presented an example of a lesson design in which the mathematization happens in reverse order: first a suitable regression formula in the form of a sum of exponential functions is determined and hereafter a two-compartmental model is developed that has the regression formula as its solution.

Distinguishing various phases and transitions in a modelling cycle does not mean that knowledge and skills must be learned and practiced separately in each sub-process. Then one runs into pitfalls of giving away formulas in a context without any explanation or of omitting the validation of a model by comparing its outcomes with experiences, observations, experimental data, or with theoretical knowledge about the phenomena to be modelled. It is our belief that students better go through the whole modelling cycle several times, first in a simplified situation (which might be a similar but simpler problem situation) and then adding step-by-step more details to the model, with the purpose of matching the model better with reality. The schematic representation of the modelling cycle can serve as a guideline for the design of the lessons: we have used the model of Blum and Leiß to design our lesson series on modelling the motion of a vertically bouncing ball, and in particular of a bouncing table tennis ball.

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MEASURING AND MODELLING A FREE FALLING AND BOUNCING BALL

The first transition in the modelling cycle of Blum and Leiß – understanding the task – means that one must come from the real world situation to a situation model. For modelling of the motion of a bouncing ball, the best thing to do is to ask oneself questions about this motion. Why in fact does the ball first fall down vertically and then bounce? What effect does the ground surface have on this motion? What effect have the choice of material, the size, shape, or temperature of the object on the motion and can this be understood? What effect had the medium in which the ball moves? Can the bounce time, i.e., the time it takes for a ball to come to rest, be computed and predicted? These questions do not appear out of the blue but originate from prior experience or by experimenting with balls. One cannot answer all questions at once and the investigation must be confined to a manageable problem situation: in our case, the study is limited to the motion of a hollow ball filled with air, bouncing vertically without spin on a hard, horizontal surface. The ball can be a soccer ball, tennis ball, or a table tennis ball; the surface can be a hard flat floor or table. Furthermore, a strategic choice of the modeller can be to first study a simpler but related problem; in this case this could be the free fall of an object.

So, let us begin the investigation of a free falling object with a video analysis of a recorded experiment and an experimental way of modelling the motion via regression. A research study of Larkin-Hein and Zollman (2000) showed that such an approach serves as an effective way to permit students to become more active learners. In addition, it enhances student motivation and attitudes as well as encourages longer time on task. Figure 3 is a screen shot of the video analysis. In the lower left window is the original video clip of the experiment in which a person standing on a ladder lets a ball fall down from a height of 4.5 m. Data collection using this video clip is a bit problematic because of the perspective distortion. Luckily, Coach provides a tool to correct the perspective view of an image plane of motion. Details about the software implementation and more inspiring examples of its use can be found in (Heck and Uylings, 2006a). The upper left window in the screen shot below shows the result of rectifying the video clip to a front-parallel view of the scene. Point-tracking makes the data collection in the rectified video clip a piece of cake. The windows to the right show the tabular and graphical results of measuring the height of the ball with respect to time. In the graph window, the (numerical) derivative of the vertical position has been computed as well: it is a straight line of which the slope is close to the acceleration of gravity. It also motivates a parabolic regression curve to fit the data.
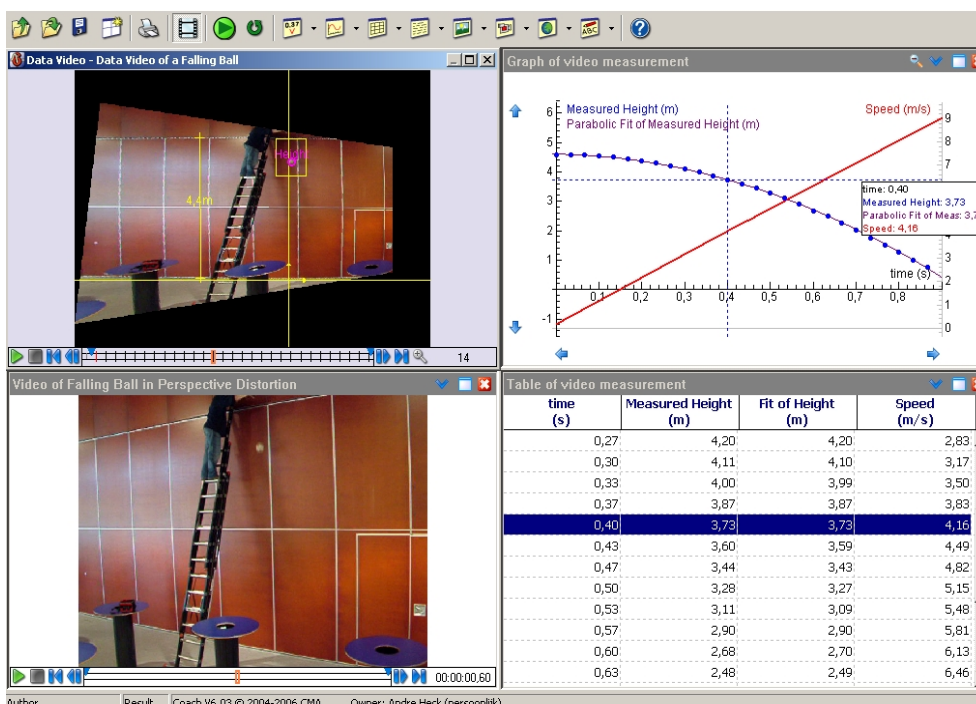

Figure 3. Screen shot of a video measurement and function fit in Coach.

Experimental modelling via regression is practical, gives excellent results, and may serve as a first run through the cyclic modelling process, but it would be nicer if one could underpin it with a mathematical model using elementary concepts of kinematics. Here, computer modelling comes into play. There exist basically three types of computer tools for simulation dynamical systems: a system dynamics approach, event-based modelling, and agent-based modelling. Coach is a hybrid system that combines a traditional system dynamics approach with event-based modelling in order to allow modellers to deal in their description of a phenomenon with sudden effects. We will see examples of this later on in this paper. Figure 4 is a screen shot of the modelling activity using only the system dynamics part of the tool. The graphical model is shown in the upper left window. The meanings of the icons are similar to those of system dynamics software like STELLA or Powersim: there are icons for state variables, flows, auxiliary variables, constants, and connectors. The graphical model represents a system of differential equations that originate from Newton's laws of motion. The diagram in the upper right corner of Figure 4 illustrates that the vertical position of the ball computed in the model matches very well with the parabolic fit of the measured motion. This concludes the second run through the modelling cycle.
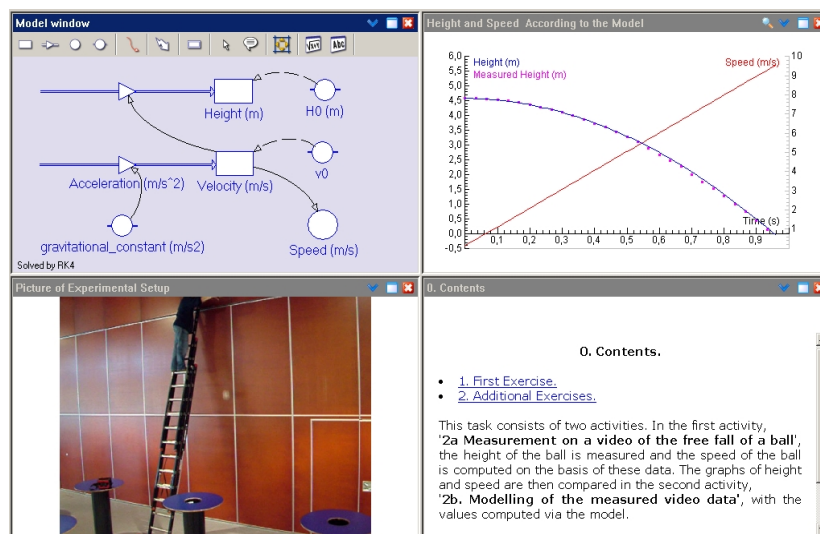

Figure 4. Computer modelling and comparison with measurement.
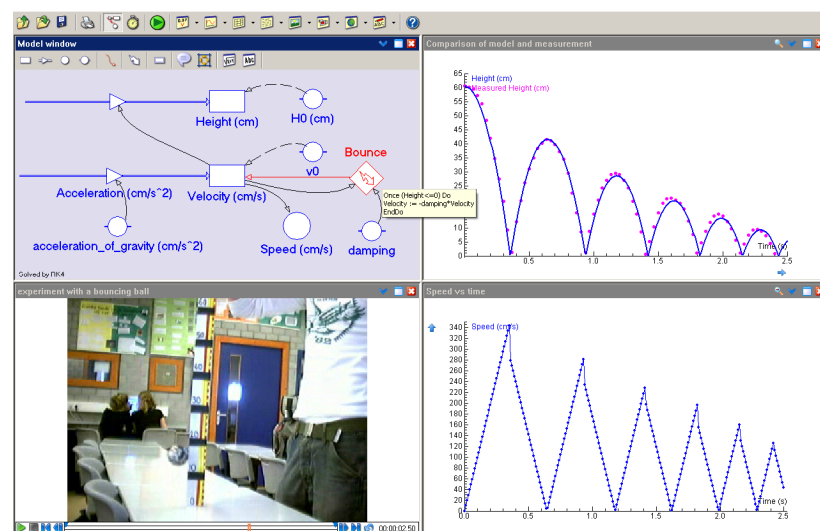

Figure 5. Modelling a bouncing solid rubber jazz ball and comparing the model with reality.

After having gone through the model cycle of the free fall of an object neglecting aerodynamic effects, we can continue with the more complicated situation of a free fall in which air resistance plays a role or with a bouncing ball for which air drag can be neglected. We select in this paper the second option; in the developed lesson series we actually treat the first option beforehand. As you can see in Figure 5, the graphical modelling window in the upper left corner is just an extension of the previous model of a free falling ball with an event icon. At the discrete time event when the ball hits the table we must update the velocity in order to change the direction of motion, i.e., to change the velocity. Let us look at the details of this event: it is triggered when the height becomes less than or equal zero. Because of energy loss due to inelastic collision – one notices that the maximum heights of the bouncing ball decrease in time – the downward velocity changes into an upward velocity and its magnitude is decreased by multiplication with a damping factor between 0 and 1. This damping factor is commonly called the coefficient of restitution. The yellow sticker, which pops up when you keep the cursor for a while on top of the events icon (the thunderbolt symbol), contains the following computer code:

```
Once (Height<=0) Do Velocity := -damping*Velocity EndDo
```

The choice of the word "Once" in the computer code instead of "If" is important. In the latter case, the condition would be checked each time when the computer program passes this piece of code and it is then possible that the sign of the velocity is forever changing, while the ball position remains negative. In order to avoid such erratic behaviour and inconsistencies an event has been implemented in Coach 6 according to the principle of software triggering: as soon as some particular condition is fulfilled, a once-only action is put in running and only at the point that this condition is not satisfied anymore, then the event can occur anew.

The graph window in the upper right corner of the screen shot in Figure 5 contains the motion graph computed via the computer model and the measured data point at the background. It shows that model graph and the measured data match well for the particular type of ball that was used in the experiment (a solid rubber jazz ball, also known as a super ball). For many a teacher and student this is often the point to stop work and continue with another topic. This is a pity because the above experimental approach to study the motion of the bouncing ball may perhaps satisfy someone who models just this particular system, but he or she must start all over again for every new kind of ball. In other words, what does the person learn from it about the motion of a bouncing ball in general? To this end, mathematics and physics must come into play, possibly in combination with a system dynamics approach that can handle discrete events. For example, the damping can be computed as the square root of the ratio of the maximum height after and before the bounce when aerodynamic effects are not taken into account. The reasoning is not beyond the level of secondary school students in physics: the speed v_1 of the ball with mass m falling freely from a maximum height h_1 when it hits the table is given by $v_1 = \sqrt{2gh_1}$, where g is the acceleration of gravity. This is true because the potential energy mgh_1 is completely transformed into the kinetic energy $\frac{1}{2}mv_1^2$. For the speed v_2 of the ball when it bounces we take $v_2 = k \cdot v_1$, where k is the damping factor, and the height h_2 to which the ball returns is determined by $v_2 = \sqrt{2gh_2}$. Therefore $k = v_2/v_1 = \sqrt{h_2/h_1}$. The damping found experimentally in this way from the measured heights in the video is 0.84; this is in excellent agreement with the damping 0.83 used in the computer model to match the data. Another question that can be successfully investigated by a student is the following: how much time does it take for the ball to come to rest? The mathematical reasoning could be that the time needed for the first and second fall are $t_1 = \sqrt{2h_1/g}$ and $t_2 = \sqrt{2h_2/g} = k \cdot t_1$, respectively. If one takes into account that the ball in the first phase only falls on the table and that hereafter it goes up and down, then it is clear that the bounce time T is given by

$$T = t_1 + 2t_2 + 2t_3 + \cdots = t_1 + 2kt_1\left(1 + k + k^2 + \cdots\right) = t_1 + 2kt_1/(1-k) = t_1 \cdot (1-k)/(1+k)$$

$$= \sqrt{2h_1/g} \cdot (1-k)/(1+k)$$

What is important about this algebraic work for education is that it is not just a hobby of the teacher, but that the formulas really help students to understand the phenomenon better. For parameter values

$k = 0.84$, $h_1 = 0.60$m, $g = 9.81$m/s^2 we get $T = 4.0$s, which is actually one second more than the time that is found in the experiment recorded in the video clip. The reason for the difference between mathematical model and experiment is that the bouncing process is more complicated, and changes when time goes on and when the last bounces of the ball take place. Another point that may surprise students is that, although the mathematical model predicts that the ball bounces infinitely many times, the total time for the ball to come to rest is finite. As a matter of fact, all these issues make the investigation not just more complicated, but also more interesting and challenging.

A DETAILED STUDY OF A BOUNCING TABLE TENNIS BALL

The first example of a bouncing ball dealt with a super ball in an experiment where air resistance could be neglected. We now look at a bouncing table tennis ball and we will see striking results. The first step in the investigation is an experiment. Figure 6 is a screen shot of the video measurement of a table tennis ball released at a height of 51 cm and then bouncing on an office desk. The video clip was recorded with a high-speed camera with a frame rate of 150 fps. The measurement of the height of the ball is automatically done via the method of point-tracking. This is easy: for each point of interest the user specifies at the start of the tracking process a template around this point that will henceforth be automatically matched in subsequent video frames. Matching takes place in a certain moving search area, the size of which is also user-definable. In Figure 6 you can see the search area around the currently measured point (in this case, the center of the table tennis ball) as a small rectangle. In this way, the coordinates of the moving object with respect to the user-defined coordinate system, the origin of which was chosen to be the point were the center of the ball is when it hits the ground, have been automatically determined in all frames of the video clip without any difficulty. Measuring in 1000 frames of a video clip can be a piece of cake!
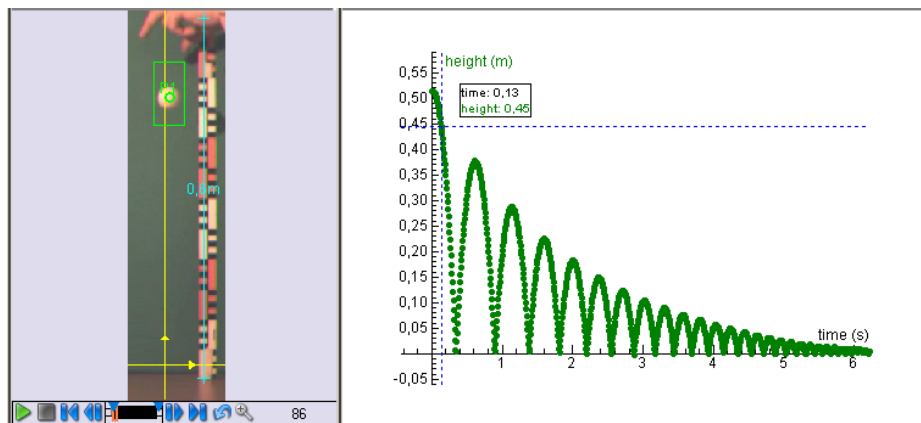

Figure 6. Video measurement of a bouncing table tennis ball via point-tracking.
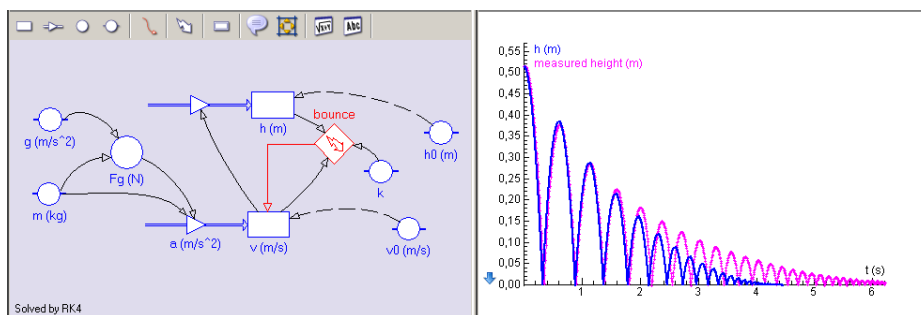

Figure 7. A less successful model of a bouncing table tennis ball.

The measured data can be used to validate the previously defined model. Figure 7 shows the result for the best guess of the coefficient of restitution ($k = 0.865$). The result of the simulation is disappointing: the model and the measurements match only for the first three or four bounces. The model predicts a too short bounce time. Maybe your first thought is that a guess for the best parameter on the basis of visual inspection is not a smart method and that the coefficient of resolution can be better determined in a systematic way. There are various methods to do this:

1. Because the video was recorded with a high frame rate, one can find a good estimate of the speed of the ball just after and before the impact. The quotient of these speeds is the coefficient of restitution for that particular bounce;
2. Look at the maximum height of the ball after and before the impact. The square root of the quotient of these maximum heights equals the coefficient of restitution in case of no air drag;
3. If one plots the time between two subsequent bounces (the flight time) against the time when the first of these two subsequent bounces took place, one gets a scatter plot of points that lie on a straight line with slope equal to $k - 1$. Note that this method can also be applied to data obtained by acoustic measurements of bounces with a sound sensor (Aguiar and Laudares, 2003),

The third method would provide an estimate of $k = 0.918$, but this value of the coefficient of restitution also does not lead to a good agreement between model and measurements. Figure 8 is a screen shot of a graphical model that not only computes the height of the bouncing ball, but also stores the computed times when the ball bounces. In the diagram to the right are shown the computed and measured scatter plot of the flight time against the time when the first of two subsequent bounces took place. In this diagram too, theory (the model) does not match reality (the measurement).
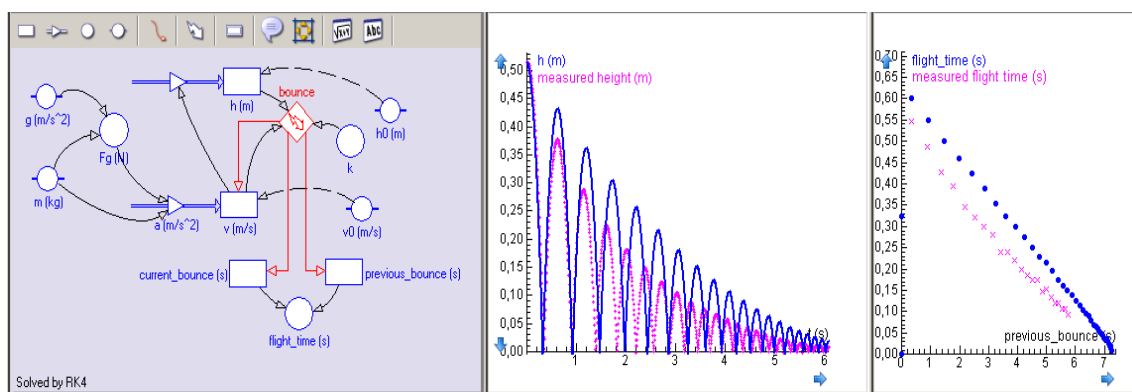

Figure 8. Comparison of a model of a bouncing table tennis ball (neglecting air resistance) with reality.

At this point students come up with various explanations: "the laws of mechanics are not correct!", "*g* changes with height", "*k* is not constant", and "air resistance cannot be neglected." The last two arguments make sense. Let us first investigate whether incorporation of air resistance in the current model makes a difference. In the model shown in Figure 9, we assume that the drag force F_d is given by

$$F_d = \frac{1}{2} \rho \cdot c_d \cdot A \cdot v^2,$$

where ρ represents the density of air, c_d the drag coefficient, A the cross-sectional area of the object ($A = \pi D^2 / 4$ for a ball of diameter *D*), and *v* the velocity of the ball. The computer model and the measurements match now much better, but theory and reality still diverge rather quickly.
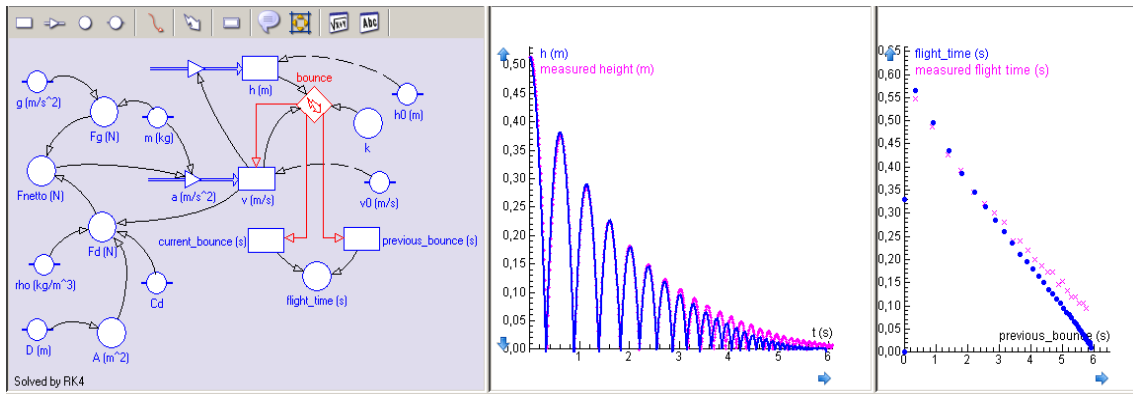
Figure 9. Improved model of a bouncing table tennis ball with air friction taken into account.

The coefficient of restitution has been considered so far as a constant that only depends on the pair (ball, surface), but this is in fact nowhere motivated. Why would it not depend on the speed of the ball when it hits the floor? In fact, for the bounce of a table tennis ball on a hard flat surface it does depend on the impact speed (Hubbard and Stronge, 2001). We have verified this by video recording of vertical bounces of the ball with a high-speed camera: for low speeds the coefficient of restitution is constant and for high speeds it linearly decreases with speed. The following relation is used in Figure 10:

$$k = 0.932 \text{ if } |v| < 1.9\,\text{ms}^{-1}, \quad \text{and} \quad k = 1.000 - 0.031 \cdot |v| \text{ otherwise.}$$
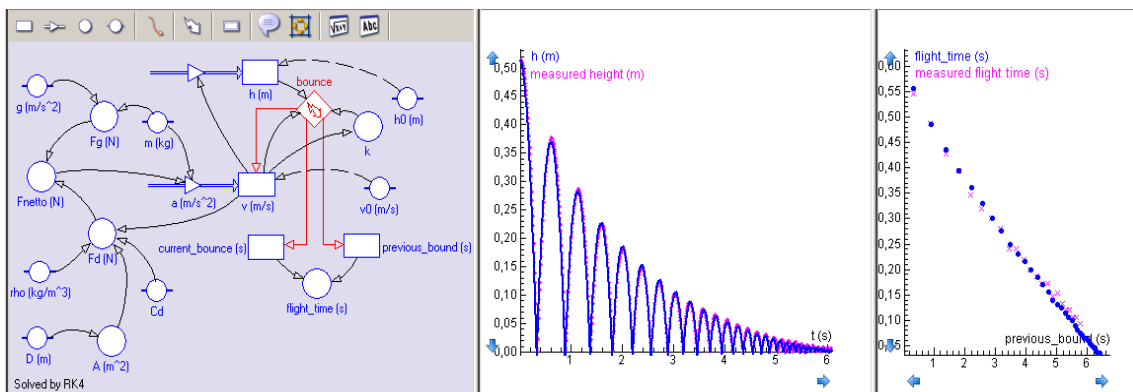

Figure 10. Bouncing ball model including air resistance and a non-constant coefficient of restitution.

Without experimentation and video recording of the motion of the table tennis ball with a high-speed camera, and without careful validation of the intermediate models we would never have gone so many times through the modelling cycle until such a nice description of the bouncing table tennis ball. This does not automatically mean that the model and all the assumptions that have been made are necessarily the only possible choices. For example, an interesting approach that avoids the use of discrete time events has been worked out by Bridge (1998): he assumed that the ball during impact phase can be considered as an ideal spring following Hooke's law. Figure 11 is a screen shot of a Coach 6 implementation of this model that illustrates a splendid match between model and measurement. The impact time can be estimated from the computed model and is about 1 millisecond. This is also in agreement with empirical results of a high-speed camera recording.
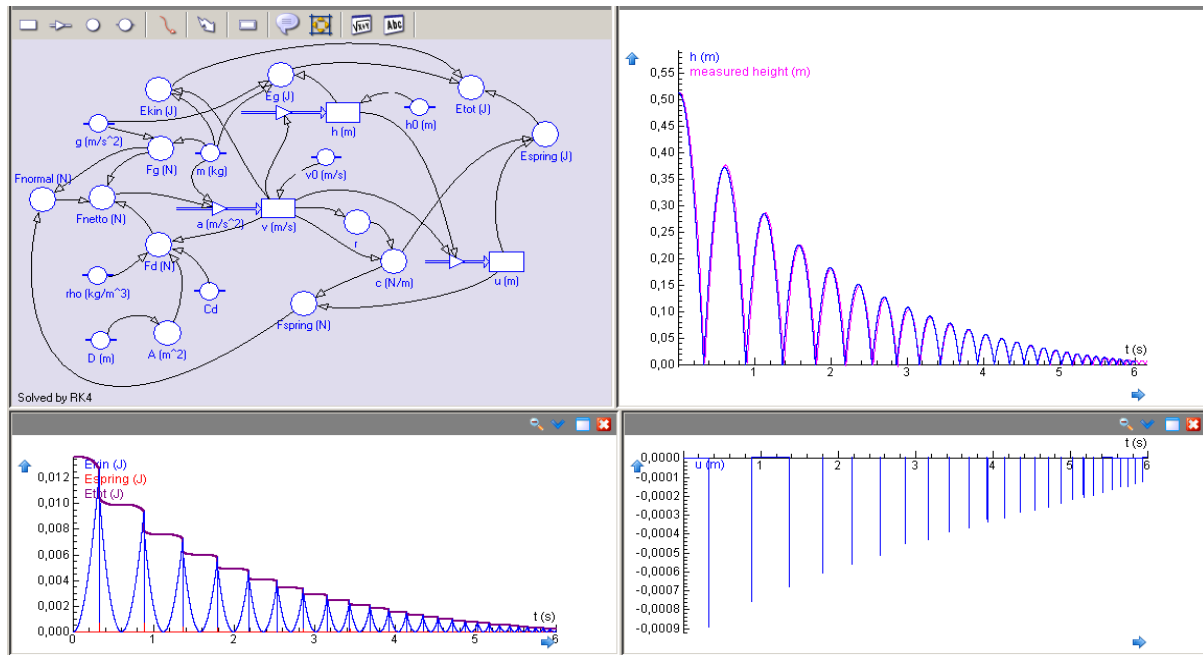
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Figure 11. Modelling of a bouncing table tennis ball as a continuous process, assuming that the motion of the ball can be modelled as an ideal spring during ball contact with the hard surface. The variable u is the computed deformation of the ball. Zooming in reveals that the computed contact time is about 1 ms.

MODELLING OF A BOUNCING BALL ON AN OSCILLATING PLATFORM

An interesting and challenging problem situation for students is the bouncing ball on an oscillating platform that introduces students also into the concept of chaos. In our mathematical discussion below we neglect air resistance but maintain the concept of energy dissipation at impact of the ball with the surface. We assume that the platform oscillates as a sine function with amplitude

$$y_{\text{platform}} = -A\varepsilon\sin\left(\frac{t}{\sqrt{\varepsilon}}\right),$$

where A and ε are positive numbers and ε is small. Then the platform is in rapid motion with a small amplitude. The following scaling of time and height − by the way, this is a much-used concept in modelling by professional modellers and is of interest for students to learn −

$$\theta = \frac{t}{\sqrt{\varepsilon}}, \quad h = \frac{2y}{\varepsilon \cdot g},$$

leads to the following dynamical system:

$$\theta_{n+1} = \theta_n + \upsilon_n, \qquad \upsilon_{n+1} = k\upsilon_n - \lambda\cos(\theta_n + \upsilon_n).$$

Here, k is the coefficient of restitution and λ is the parameter given by $\lambda = (1+k)\cdot A$. This dynamical system can behave in many different ways. For example Figure 12 illustrates that the simulation of the computer model for parameter values $A = 5, \varepsilon = 0.001$ leads to periodic behaviour of the ball. The diagram windows in Figure 12 show different time intervals or zoomed-in versions.
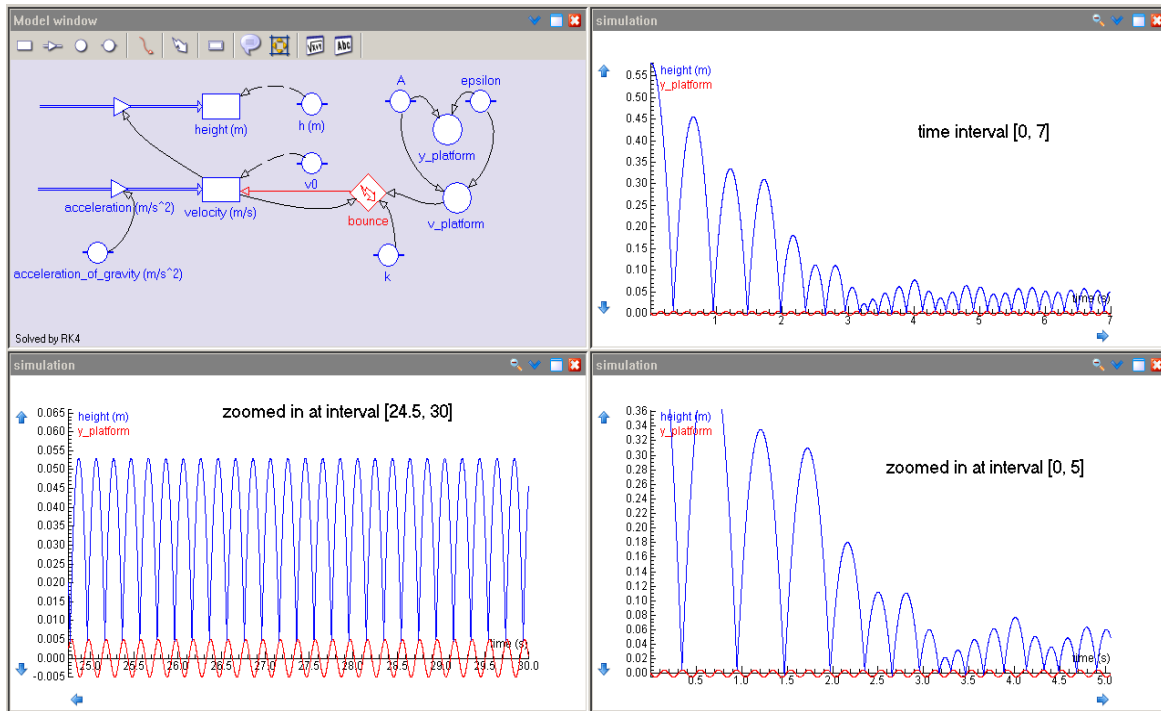
Figure 12. Periodic long-term behaviour of the bouncing ball on an oscillating platform.

Figure 13 illustrates chaos: the behaviour of the system is completely different when ε equals 0.00099: the ball in the end follows closely the motion of the platform. Animations (release 6.3 of Coach) linked to the computer model visualize these phenomena in an appealing way.
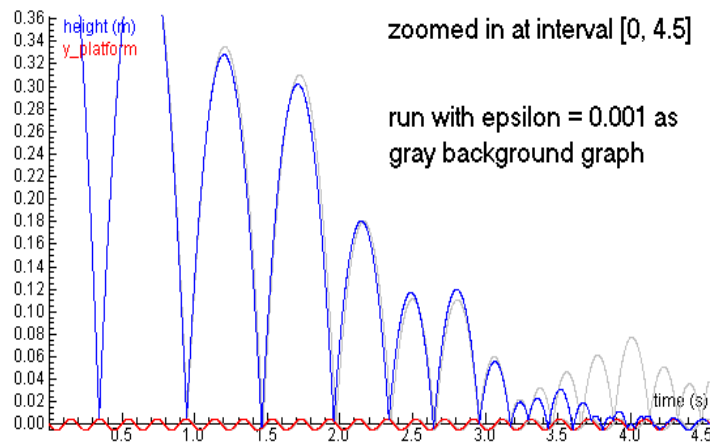

Figure 13. The bouncing ball on an oscillating platform that stops bouncing after a short time.

CONCLUSIONS

The progressive aspect of modeling, visualized by a modeling cycle like the one of Blum and Leiß, is a pointer to a suitable manner to introduce it to students: it seems best not to let them construct out of the blue some well-functioning model, but to let them first start with modeling of a simple but related problem situation, or to let them first improve an existing model by changing or adding details. Here it is important that students can compare the results of the computer model with real data, preferably collected in an earlier measurement activity. Confrontation of a model with reality turns (graphical) modeling not only into a fun way of learning, but also makes it exciting, challenging, and concrete work for

students. Our experience is that this is practicable, certainly when this validation of models is treated repeatedly over a long period. Our lesson series on bouncing balls does not necessarily have to be done in one row but can be spread over a period of months or even years. For example, the parts about free fall with or without air resistance can be done in one school year and the study of bouncing balls in another year, or as part of an independent practical investigation task.

Using discrete events, modelling of complex system dynamics is not beyond most of the students' abilities anymore. It really widens the scope of computer modelling in education. In our case, it has led to a realistic, physics-based model of a bouncing ball, the simulation of which matches in an excellent way with the experimental data at the beginning of this process. Heck and Uylings (2006b) have reported similar success of investigating the motion of a yoyo via video analysis and graphical modelling.

What is important for the kind of modelling activities discussed in this paper is that students have at their disposal a versatile computer learning environment with integrated tools for measurement, modelling, and mathematical analysis. Coach 6 is such an environment: it provides a graphical, system dynamics based modelling tool that allows its user to deal easily with discrete time events and, in addition to measurements with sensors, it offers a modern video analysis system with tools for perspective correction and point-tracking. This last tool is convenient when high-speed video clips come into play and manual data collection is laborious, time-consuming, and error prone.

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