# DESIGNING A TASK FOR EXPRESSING RANDOMNESS

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ABSTRACT
This paper describes how children express their ideas for randomness in a two-dimensional continuous space, through tools for directing and redirecting the simulated movement of bouncing and not bouncing balls. The paper reports a part of the findings of a study in which children aged between 5.5 and 8 years old engaged with a game-like environment. The findings reported in the paper are part of a broader study that adopted a strategy of iterative design, in which the computer-game was developed alongside the gathering of evidence for children's use of the tool. In response to a range of tasks, children manipulated the sample space in ways that generated corresponding outcomes in the game. The episodes of the case study presented in the paper indicate that in the medium of expression provided by the game and the evident connection between sample space and global outcomes, children constructed novel meanings of randomness.

KEYWORDS
Randomness, computer-game, probability, sample space, iterative design

## BACKGROUND AND AIMS

In the field of learning probability, there appears to be a gap between early intuitions of probability formed from everyday life and the knowledge gained from formal learning. This gap can be explained as 'misconceptions' (for example, Kahneman & Tversky, 1982 ), or, according to diSessa (1988) incorrect connections between pieces of probabilistic knowledge. Yet, despite the seminal contributions that Piaget & Inhelder (1975), Kahneman & Tversky (1982) and others made to research on randomness, little attention has been paid to the tools that learners have available for expressing themselves. Our starting point is that by designing tools that are specially designed for expressing randomness and chance, and letting learners express their ideas with them, learners may do things that cannot be predicted simply by misconceptions or by stages of thinking.

Wilensky (1995; 1997) has based his work on the conjecture that both the learner's own sense making and the cognitive researchers' investigations of this sense making are best advanced by having the learner build computational models of probabilistic phenomena. He shows that through building computational models, learners can come to make sense of core concepts in probability such as normal distribution. Learners are supported in building and developing their mathematical intuitions and, through this construction process, mathematical objects are seen to be more concrete as learning progresses. Pratt's (1998) study, also showed that by using a computational system, children managed to make sense of local and global probabilistic resources or meanings, referring to *local* resources of randomness as those based on experiencing the outcome of individual events, and of *global* those focused on an aggregated view of probability. According to Pratt (1998; 2000) local resources in probability are characterised by the fact that the next outcome is unpredictable, there is irregularity and there are no apparent. Global resources have the following characteristics: the proportion of outcomes for each possibility is predictable, the proportion will stabilise as an increasing number of results (large numbers) and there is control through manipulation of sample space. Moreover, Pratt describes the development of a computer-based domain within which children (aged between 10 and 11 years) manipulate stochastic gadgets, representing everyday objects such as a die, a coin, a lottery and a set of playing cards. Individual learners were put in situations where they can express their beliefs in symbolic (programming) form and articulate the beliefs that they hold, construct, and reconstruct them in the light of their experiences.

In the above cases the computer environment acts as a window. 'Window', as this is defined by Noss & Hoyles (1996), refers to the fact that although you cannot observe a learner's thinking directly, you are able to study and analyse the learner's actions as they come to the surface by playing with the situations that the computer environment provides. Noss & Hoyles claim that the theory behind the transfer methodology in opening windows on microworlds presupposes that there is a single path or at least a single right answer that is independent not only of the setting, but the intellectual and physical resources, which are available for its solution.

The aim of this paper is to describe the design of a tool constructed simultaneously to afford expressive power to children aged six to eight in the domain of probability, and to provide a *window* (Noss & Hoyles, 1996) on that thinking is developed. This research aims to study the ways in which local resources of randomness, based on experiencing the outcome of individual events, can be developed into global understandings that focus on an aggregated view of probability (e.g. probability of an event). The broader study also assesses whether and how the explicit linking of local and global resources via a rule-based system, assists in effecting this evolution. However, in this paper, we will focus on design and on the ways in which our design-criteria afforded children to express their ideas of randomness.

## OUTLINE OF THE DESIGN APPROACH

The study sets out an approach to iterative design in a rule-based system medium, *Pathways*[1]. Pathways is a graphical iconic interface (written in Imagine, Logo[2]), which allows children to build and modify rules. Pathways was chosen mainly for two reasons: (a) the children's opportunity to manipulate, especially the opportunity to manipulate the sample space, and (b) the 'showing and reacting to messages' function that acts as a connector between the elements of the task.

In Figure 1, we outline an example of how the user can manipulate different elements of a game in Pathways.

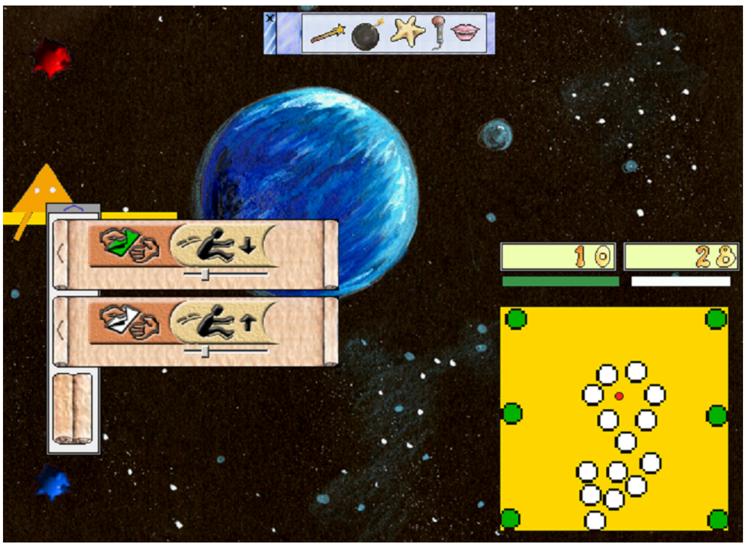

Figure 1. An example of manipulations in Pathways

[1] Pathways was funded by the European Union as part of the *Playground Project*, Grant No. 29329. See http://www.ioe.ac.uk/playground/

[2] http://el.media.mit.edu/logo-foundation

For example, in the rectangle in the bottom right corner there are some balls that ‘live’ in a ‘lottery machine’, controlling the movement of the space kid. Particularly, the lottery machine acts as a sample space of the game. So that, when two balls collides, an event is triggered which moves the space kid. The direction of the movement of the space kid depends on the colour of the ball. The small rectangle at the upper centre of Figure 1 is the toolbox, with which by using the wand you can copy any object in the game and by using the bomb any object can be destroyed. So, children can use the toolbox to add or take away balls and move the balls inside the lottery machine in order to increase or decrease the probability of a ‘hit’. They can also move the objects, depending on the goal of the game, for example they can increase or decrease the distance between the planets or create new ones, change how far an object can jump, or create new objects and give them new rules.

As we stated earlier, an important design criterion was to choose a tool that worked with rules affording an opportunity for children to understand how the elements of the game are connected and moreover, to link local and global events (see Figure 2).


Figure 2. An example of linking rules in Pathways

Figure 2 shows an example of how Pathways works. Using the library of stones the user can make rules in order to connect the one object to the other. So, when, for example, the bouncing ball collides on a coloured, *blue or a red* ball, this ball can show a message to another object, for example the space kid, to react, move a step downwards and play a sound. The rule in Figure 2 can be read as ‘when I touch anything, I bounce it off and I show a *blue* message’ and the reaction of the other object is ‘when I see a *blue* message, I move a step downwards and I play a sound’ (the colour of the message can be chosen from a library of colours that the software provides).

Our approach followed that of iterative design, as defined by diSessa (1989)

> ‘one must carefully observe and document the activities of children in prototypes of the proposed microworlds…including some sense of the span of conceptual states that children might be in.’ (p.216-127).

The iterative process facilitates, through the gradual refinement of computational tools, an increasingly fine focusing on the primary issues involved in this restructuring process. In the particular study, the refinement of the game and its tools, by taking care the previous iteration of children working with it, gave insights to the next design of the tool. Bearing in mind that Pathways was developed as the study grew, iterations also gave feedback to the software itself, by suggesting new functions in the next version that would work for the task as next iteration as well.

## THE DETAILED DESIGN OF THE GAME

A space kid moves upwards and downwards on a line (see Figure 1). A lottery machine, in which a small white ball bounces and collides *continually* with a set of blue and red balls, controls the movement of the space kid. The control of the movement of the space kid is achieved by the showing and reacting to messages, as is described to a previous paragraph. The movement of the space kid and the continuity of the small ball in the lottery machine were designed in order children visualise the global outcomes of the game. Children could change and manipulate a number of aspects to construct their own sample space. They can change: the number, the size, and the position of the balls in the lottery machine or they can create new objects with their own rules. Collisions with the *blue* balls (light grey in the figures) add one point to the *blue* score and move the space kid one step down the screen. Whilst individual collisions can be seen as single trials in a stochastic experiment, the totality of these movements gives an aggregated view of the long-term probability of the total events. In order to explore the connections children make between fairness and randomness, we began with a situation in which the children had to try to make the space kid move around a centre line in order to construct a 'fair sample space'. The following paragraphs describe a case study of a young girl who is playing the game and constructs understandings of randomness.

## A CASE STUDY: SOME ASPECTS OF RACHEL'S THINKING ABOUT RANDOMNESS

In keeping with the iterative design approach, the game was developed alongside the gathering of evidence for children's use of the tool. All interviews were videotaped and transcribed. Rachel (aged 7 years and 3 months) is one of 23 children, aged between 5.5 and 8 years, involved in the final iteration. The children worked with the software individually for between 2 and 3 hours.

Most of the children initially focused on the movement of the white ball inside the yellow square. Children seemed to be interested in watching the *arbitrary* movement of the white ball, which moves around (apparently) arbitrarily. Rachel similarly initially focused on the movement of the white ball. She concentrated on the sample space, trying to impose on the white ball some kind of determined and predictable movement. However, the continual movement of the white ball did not make it possible to find a predictable movement of the ball. The following snapshot describes Rachel's first reaction, after she made her first change in the sample space.

Rachel: The score will change, now. The blue (the light grey ball in Figure 3) will get fewer points because the white ball hardly ever goes to the corners.

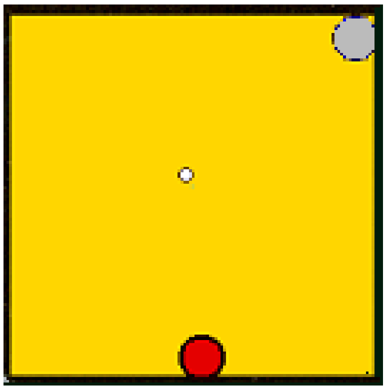

Figure 3

Researcher: Why is that?

Rachel: The white ball moves like this here and there. It moves up, down, right, and left. (S*he is indicating the movement on the screen, by avoiding the corners).* I don't think that it goes to the corners… Let me see.

She starts the game.

Rachel: You see, I was right! Oh! No! One point for the

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| | blue score, another one, oups! |
| Researcher: | How does the white ball move? |
| Rachel: | It is moving where it wants to…I mean if it is here (*she is pointing on the screen)*, in the middle, it might go here and here, and here, everywhere…look we have equal points now! |

At first, Rachel explained away the movement of the white ball. She described the movement of the ball as 'deliberately' avoiding the corners, basing her decision on her idea that the white ball 'hardly ever moves to the corners of the square'. In fact, a more general observation is that, like other children, Rachel's explanation of randomness focuses on the movement of the white ball, rather than, for example, on when the coloured ball is touched and with what frequency. But, according to the above episode, as the white ball moved in the lottery machine without stopping, she expressed that movement as the ball 'moving where it wants to'.

It seems that 'continuity', in the sense of continuous movement of the ball was important in helping her link events in the sample space to the global outcomes; that is the aggregate view of the movement of the ball seemed to help Rachel link the local meanings for randomness - the movement of the white ball- with the global understandings and long-term behaviour - the movement of the space kid. Figure 4 shows how the medium helped Rachel to link the different levels of randomness.

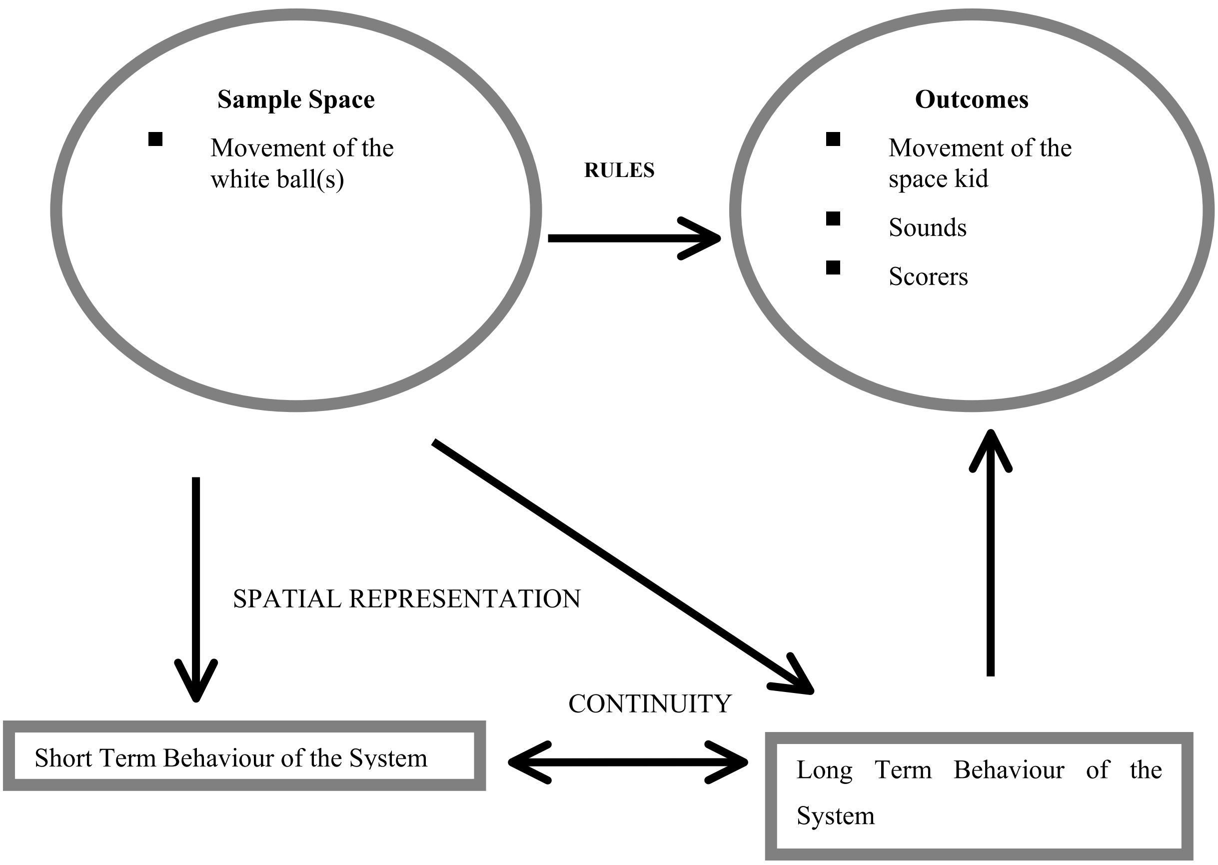


Figure 4. How the medium helped Rachel to link the local and global levels of randomness

Figure 4 illustrates how the rules of the game linked the sample space to the outcomes. Rachel linked the movement of the white ball with the movement of the space kid, the sound in the movement and the scorers, which represent the outcomes of the game. The spatial representation and the continuous movement of the bouncing ball made it possible for her to see the movement of the white ball, not only as a short term behaviour of the system, but also as a long term behaviour. The spatial representation in the game connected also the sample space in a short term and long-term behaviour. This played a major role for Rachel to connect local and global events. A snapshot that describes a shift from local to global events is the following:

Rachel: I will destroy some balls… (see Figure 5).

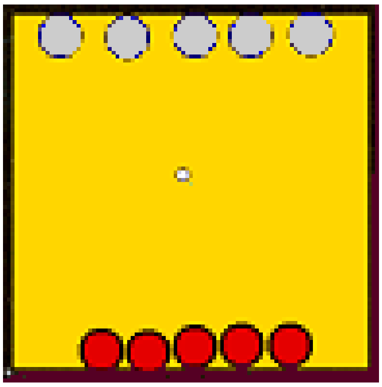

Figure 5

She starts the game.

Rachel: Did you tell me to be on the yellow line? (She laughs). Oh… it moved upwards to the blue mine.

Researcher: Why was that?

Rachel: Because the ball moved like that and touched the blue balls… I don't remember very well how was moving…When it touched a ball it changed where to go…I don't know how it moved around.

The simultaneous visualisation of the local and global events that the medium provided seemed to help Rachel to link them. It seemed also that continuity in the lottery machine afforded her not to stay focus on the short-term movement of the ball. Rachel tried first to base her decisions on the short-term movement of the bouncing ball and guess how the ball would move and which balls would touch more frequently. But, continuity in the medium made her 'not remember' exactly how the ball was moving and this to be the reason for not knowing how the ball was moving around.

**DISCUSSION AND CONCLUSIONS**

As the literature describes (see for example Jones, et al, 1999), Rachel was expected to look for patterns in order to explain the random behaviour of the small white ball and the outcomes of the lottery machine in the game. Although that was her first attempt, after playing the game for a while, the situation changed. We conjecture that this change of focus occurred because of the feature of the microworld. We can conclude that the continuous movement of the bouncing ball and the visualisation of the rules of the game, did not allow Rachel to focus for a long time on looking for rules of randomness.

In general the children of the study had a strong urge to control an unpredictable situation and find ways of predicting a result. The children's attempt to understand randomness by using tools of the game shifted their focus from looking for ways to control random behaviour, towards ways to control the events by making changes in the way the sample space 'worked'. Rachel's case illustrates how children at first tried to find how randomness works by isolating it from the random environment.

Probability can be described as an intellectual tool for describing and predicting unpredictable behaviour; likewise for young children in this study, constructions in the lottery machine were an attempt to control an arbitrary behaviour. The children's constructions show that their thinking shifted from looking for rules in randomness to control the random behaviour by looking at the events in their sample space.

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