ANOTHER WAY TO TEACH NON-CONGRUENCY

Jiří Vaníček

ABSTRACT

New technologies in education allow and sometimes demand changes in school curricula. They enable children to look into those parts of science, which have been forbidden to them because of too high a level of required imagination or too difficult calculations.

This article deals with use the software of interactive geometry Cabri in education in the topic of congruent geometrical transformations. Some children don't see congruency as mapping of the subject to the other one, but as a movement of the same object to any other place. One of the reasons for this could be that children don't have any experience with some understandable non-congruent transformation. Using manipulation and experiment with one of sensibly behaving non-congruency, the child can gain enough examples for understanding the conception of non-congruent transformation and then of congruencies.

We have chosen cylindrical mirroring as a convenient non-congruency because of its good real representation by a physical experiment. We prepared and tested a teaching project with a group of 12-years-old students. Children first conducted the real experiment with the cylindrical mirror, then they simulated the mirroring by using interactive geometry software and checked printed examples by the mirror and then made their own pictures on the computer screen using pre-defined macros or by trying their own constructions of cylindrical mirroring The dynamical functions of the provided software gave them knowledge about how cylindrical mirroring "works".

KEYWORDS

CAL in mathematics, dynamical geometry software, curricula, constructive approach, non-congruencies, cylindrical mirroring, Cabri Geometry

A positive demand of using information and didactical technologies in education into quality of teaching is known and accepted very well in didactical and psychological aspects (visualization, experiment, constructivism, immediate feedback, change of the role of teacher and student in the classroom, learning to work in team …). Using technologies has a big demand on school curricula changes. We have gained 20 years of experience with this by introducing electronic calculators into Czech schools – in an relative short time the educating process in mathematics changed a lot: automatisation of calculus stimulates the introduction of student thinking curricula.

Computers offer the opportunity to bring new quality into mathematics teaching. Tools of interactive and dynamic geometry, like Cabri, Sketchpad, Cinderella or Geonet, can show children some parts of the hidden world of mathematics on the screen. Students can see and make geometrical constructions more quickly, exactly, in a more illuminating way and dynamically. By gaining a large amount of experience, of separated mental models of observed subject or phenomenon in a relative short time, students are enabled to build universal model more quickly. A student can concentrate much more on creating mental operations like

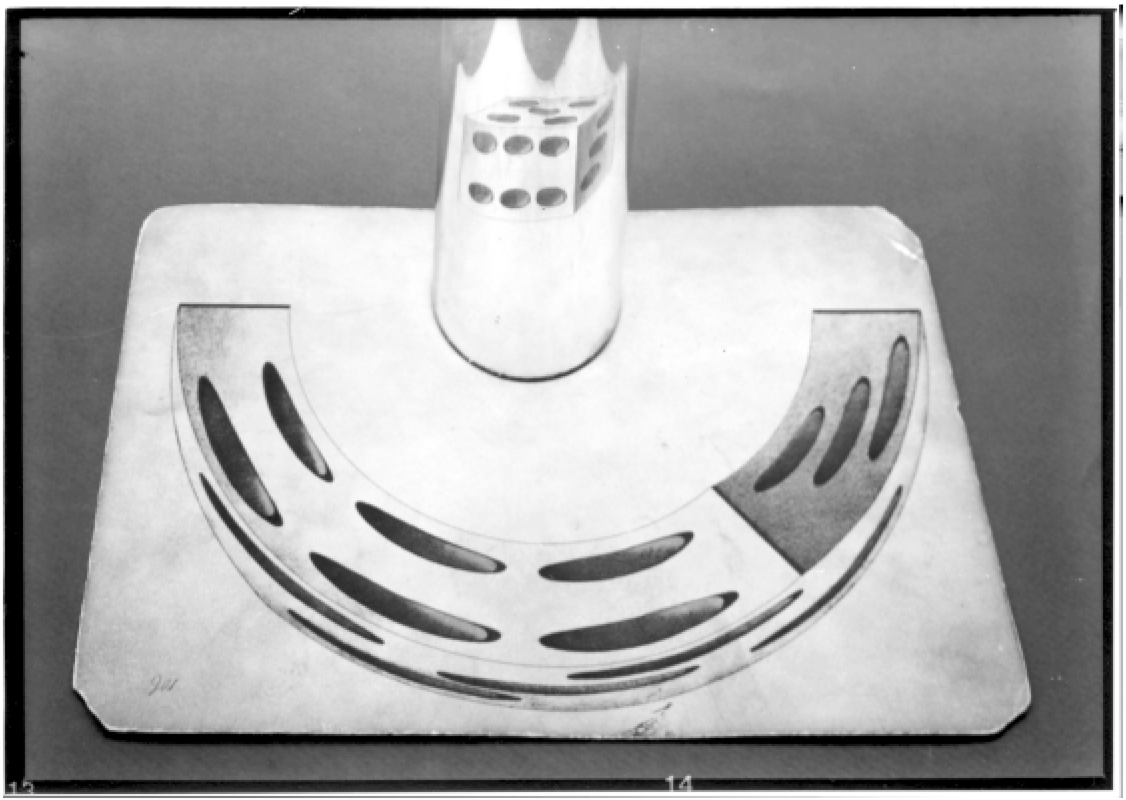

Fig. 1 - cylindrical mirroring as a museum exhibit

abstraction, induction, analysis, comparison, synthesis by releasing of actual memory capacity. Students are not heavy-laden by the necessity to imagine and visualize the situation, they are not disturbed by the technical problems of drawing on the paper.

A computer can help especially in these parts of maths, where problem understanding depends on necessity to build the right conception. One of the examples, which this article deals with, can be the concept of congruency, congruent transformation. Many teachers experience that especially average or poor students don't understand the concept of congruency, they cannot map some geometrical subject to the other one in difficult situations (for example in reflection where the axis is neither horizontal nor vertical), they don't see congruency as transformation.

This problem can be demonstrated by this example:

a) A rectangle and axis are given. Which position of the axis the rectangle will transform to itself in reflection?
b) A triangle and axis given:. Which position of the axis the triangle will transform to itself? Which shape of triangle allows to manage the example to be soluble?

We can see that even very good fourteen-year-old students often have problems with solving this theme. They don't understand symmetry of the object as a condition for laying objects and its image at the same place. It may be caused by using this teaching method. Pupils are introduced into congruency by comparing two objects practically – by moving a transparent slide on which the shape of one object is drawn. The child can understand transformation so it is only moving the same one object to the other place, not the relation between two objects.

Lack of other examples explaining congruencies can lead students to creating wrong conceptions of geometrical transformation – children don't learn sufficiently about non-congruencies, where the two objects are different, they have different growth and shape and tell children: we are two, not only one moved object.

Another example how to create right students conception of congruencies can be introduced with non-congruencies. Technical problems avoiding teachers to teach the topic of non-congruencies are over because computers and interactive geometry software can help by constructions, by visualization, by getting information about behaving of objects. Students can drag triangle or axis on the screen by mouse and observe changes of position of transformed triangle in created construction of reflection of triangle. They can also do this in prepared construction of axis affinity of triangle without any knowledge about it, they can gain determine basic information from the figure on the screen.

Fig. 2 "teacher portrait" - a pupil computer experiment with an invert mirroring in Cabri Geometry

We only have to find „good“ non-congruency, that means real, understandable, in real life observable phenomenon. For example the disadvantage of axis affinity, that could show non-congruency of two objects very well, is the difficulty with searching its real model. Without projection, a common student can rarely see and identify affinity out of school. The author thus investigated such non-congruent transformation, which is more real. The author thinks that cylindrical mirroring, transformation known from physics from the phenomenon called light reflection in a cylindrical mirror, can play this role of "real non-congruency".

What is the physical core of the phenomenon of this anamorphic transformation? A ray of light flowing out from the object, then falling on a cylindrical mirror and reflecting to the eye of the observer (staying in infinity). This experience deceives this observer because the light moves directly and observer can see an image of the object in a mirror. Depending on the shape of the mirror the object and image aren't congruent. (fig. 1)

We can construct a mathematical model in a perpendicular projection of this transformation to the plane perpendicular to the axis of cylindrical mirror on the computer screen.

This projection of the mirror is modelled by a circle *k*, in which the mirror is staying on. The direction of observing (that means direction of reflected light rays) is modelled by ray *p*. The object (picture painted or printed on the paper which the mirror is staying on) is the object *O*, its image is called *O´*. We can describe this cylindrical mirroring by equations *x´=k.x, y´=k.y*, where the number *k* is

$$k = \frac{\left(r + 2b + 2\sqrt{r^2 + 4br + 4b^2 + 3a^2}\right).r}{4a^2 + 4b^2 + 4br + r^2}$$

x,y are coordinates of object (of some point on object), *x´,y´* are coordinated of image of the object, circle k has the radius *r* and the central point in the beginning of coordinate system. The observing direction is on the negative axis *y*.

A real experiment enables a student to understand a computer visual model of this example. The teacher has to prepare a cylindrical mirror. If you don´t have it in a school collection you can make it easily from PET bottle covered by mirroring self-sticking wall-paper available in every stationery shop - fig. 3. And print the computer model of the situation visible on the screen on the paper so that diameter of the circle *k* is exactly the same as the diameter of the mirror.

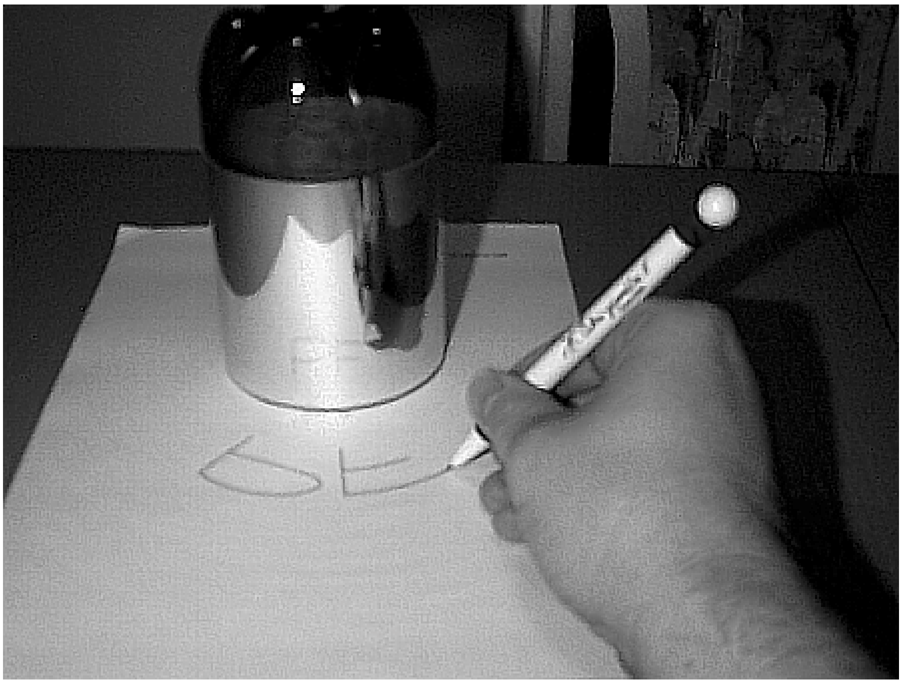

Fig. 3 Photograph of a student experiment with invert image designed using the computer and printed

Pupils can experiment: if they place a mirror on the paper at the circle, they can see an image of the printed object in the mirror, as the computer had shown before. The printed image is exactly at the same place as observer can see it in the mirror. This experience is sufficient for children to understand computer models and then they can try to experiment with cylindrical mirroring in a computer. Students can change a shape of an object and watch the image. Thus print their own situation and check it with a real mirror once more, usually it is not necessary. Students can develop their creativity by creating their own objects, then they can test similar transformations of a point with a teacher support. (fig.2, 3)

The program Cabri Geometry includes useful and powerful tools for these activities (locus, macros) which enable transformations of relatively difficult images. You can try to construct images of any object because the construction is very simple: when the point moves on any object, its image moves too. Locus of this image point is the sought image of the object. While several graphical software products are able to show images of any object in a cylindrical mirror, we have not found any software which could execute to invert a transformation to this one. For example in 3D-modelling software you can create a cylindrical mirror and reflect in it a square laying before the mirror, but you are not able to construct or define object laying before the mirror reflecting to square. In dynamic geometry software, especially in Cabri, it is possible.

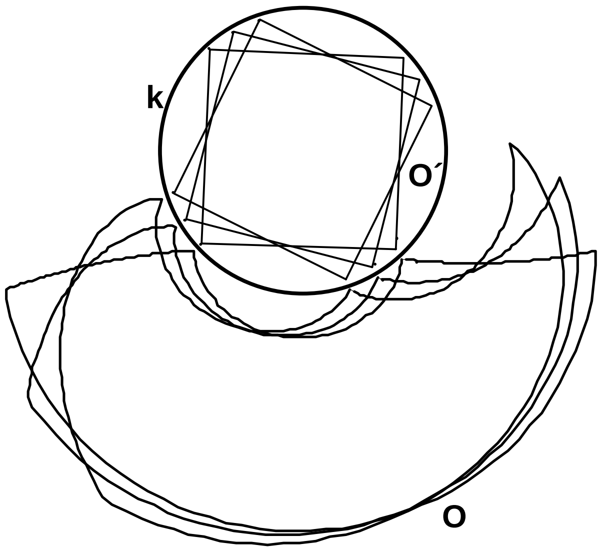

Fig. 4 Invert cylindrical mirroring. Phased motion of object O and its mirrored image - the square O´

Further experiments are available: change of shape or object transposition (fig. 4), change of mirror radius, of vector determining observation direction. A very nice activity seems to be guessing a shape of a subject. A student constructs an invert transformation to the mirroring so that he can create the image first and then the computer will transform this image to its object. After hiding an initial image can guess the other shape of the image and after printing it out they can practice an experiment and check their guesses. The correct solution will appear on the mirror. Then a teacher can give difficult exercises concerning a printed matter to best students. They can only guess without using a computer (fig. 5). It could be a small training of topology.

We can demonstrate relations between congruencies and non-congruencies to the students knowing the concept of infinity. We can prepare such constructions of cylindrical mirroring where radius of the circle can grow to the infinity. Then circle will move to a line and cylindrical mirroring will fluently change to plane mirroring and our computer model to reflection, to congruency. Two related objects can be congruent and non-congruent only by dragging the circle *k*. (fig 6)

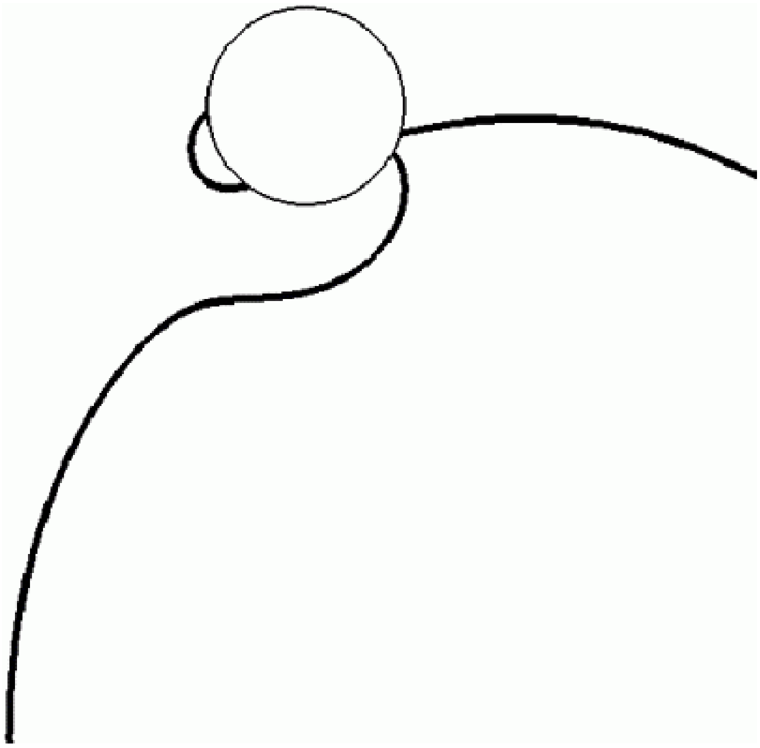

Fig. 5 Guess which conic could be image of this curve in cylindrical mirroring.

We are experienced to do this project with the age 12, 14 and 17 (and also student teachers at faculty) and after observing twelve-year-old pupils we can say that children almost have understand the concept of non-congruent transformation and they were able to follow and practice activities describing above.

We have other experiences that demonstrate that teachers have more problems with preparing and executing these untraditional projects and using this software in a classroom or in a computer lab than students have. With respect to this fact we try to educate new mathematicians at teacher training faculty so that they are prepared for future when the financial and technical conditions enable them to include this topic into new mathematics curricula.

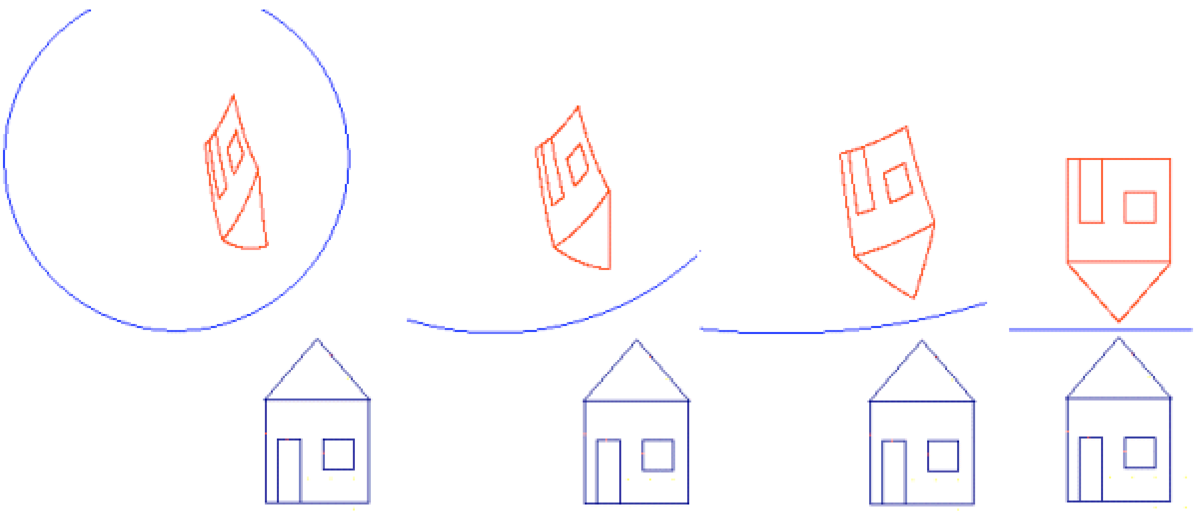

Fig. 6 relation between congruency and non-congruency in the example of mirroring

REFERENCES

Gremillion, D., Keyton, M.: Cabri Geometry II - Guidebook for Windows, Texas Instruments Incorporated 1997

Heid, M. K.: The technological revolution and the reform of school mathematics, In: American Journal of Education, Vol. 106, No. 1, University of Chicago 1997, pp. 5 - 61

Hejný, M. a kol.: Teória vyučovania matematiky 2, ŠPN Bratislava 1987

Resnick, L. - Johnson, A.: Intelligent machines for intelligent people: cognitive theory and the future of computer-aided learning, University of Pittsburgh Press, USA 1988

Schumann, H.: Schulgeometrisches Konstruieren mit dem Computer, J. B. Metzler Stuttgart 1991

Vaníček, J.: Experience of the Preparation of Mathematics Teacher Students With CAL of Constructive Geometry, In: proceedings No. 1 MEDACTA99, Nitra, SK, June 9.-11. 1999

Vaníček, J.: Počítač jako nositel nových trendů ve výuce matematiky, In: Matematika, fyzika, informatika Vol. 10, No. 6, Prometheus Praha 1999

Vrba, A.: Geometrie na počítači, textbook to the course TTT Europe, 1999

Jiří Vaníček
University of South Bohemia
Faculty of Education
Jeronýmova 10
371 15 České Budějovice
Czech Republic

E-mail: vanicek@pf.jcu.cz